Electrical permittivity shape identification using electrical capacitance tomography data and level set formulation

M. Soleimani  
The University of Manchester  
M.Soleimani@Manchester.ac.uk

A. Movafeghi  
Sharif University of Technology  
amovafeghi@ieee.org

Abstract: Electrical capacitance tomography (ECT) seeks to image the electrical permittivity distribution. ECT has potential applications in monitoring of two-phase flows. The image reconstruction in ECT is an inverse medium problem and it is ill posed and nonlinear. Two-phase material reconstruction is a shape identification problem. Current states of the art techniques are pixel based image reconstruction methods, which are not effectively formulating the two-phase property. In this paper we present a new interface based shape identification program using level set formulation. The level set method is a powerful technique in tracking interface propagations and has many applications including image processing. In this paper we study application of the level set method for a shape identification problem in ECT. The inverse boundary value problem of the low sensitive capacitance tomography imaging can be solved efficiently using level set method. Shape reconstruction method and especially formulation based on level set function can provide enough information to identify the object to be imaged. The technique reconstructs the interface between two phases. Main contributions of this paper are to introduce level set method to the inverse ECT problem and also reconstruction of the permittivity shape using experimental data.

Keywords: Electrical capacitance tomography, ECT, image processing, shape reconstruction, level set method

1 Introduction

Electrical capacitance tomography (ECT) has many potential applications in monitoring of two-phase permittivity application [2]. The most important applications are in oil industry such as oil and gas separation. The task of image reconstruction for ECT is to determine the permittivity distribution and hence material distribution over the cross-section from capacitance measurements. Ill-posedness of the inverse problem making the solution sensitive to measurement errors and noise. Even a small amount of noise in the data can cause artifacts in the reconstructions that might render them useless for practical purposes. In many applications the structures, which are sought, are not necessarily smoothly varying and might have a high contrast to the background parameters, the reconstruction of blocky or discontinuous images might sometimes become more interesting. The regularisation technique can be used to incorporate prior knowledge to stabilize the solution of the inverse problem. In some applications, the goal of the imaging system is the recovery of information concerning the number, shape, size, and perhaps contrast of a collection of anomalous regions. In two-phase ECT problem these information are adequate to identify the object. In this paper, we concentrate on two-phase material reconstruction, but multiple level set methods can be applied to three and more phase materials. In this paper we compare the result of the level set based formulation with more traditional pixel based image reconstruction. Similar works in this area are applied to the simulated data [1],[3]. In this paper we present some of the first experimental reconstruction. We have implemented FEM forward solvers; image based methods and shape identification techniques for electrical capacitance tomography as well as some other electromagnetic tomography techniques [4]. The technique
implemented here can easily be used for similar modality electrical resistance tomography [5].

2 Purpose of the research

The image reconstruction in ECT is an inverse medium problem and it is ill posed and nonlinear. In some cases the objective is to reconstruct two-phase material. This is a shape identification problem and an inverse boundary value problem. The Level set method is a powerful technique in tracking interface propagations. In some medical and industrial imaging techniques the level set method has been used for shape identification and image processing. In this paper we study the use of the level set method for the shape identification problem in ECT. The inverse boundary value problem of the low sensitive capacitance tomography imaging can be solved efficiently using level set method.

3 Innovations

In two-phase material imaging the goal is to recover information concerning the number, shape, size, and perhaps contrast of a collection of anomalous regions. There has been growing interest in the development and use of geometrical inversion methods, which move away from the estimation of a dense collection of pixel values and concentrate processing resources directly on the recovery of information regarding anomalies. We reformulate the problem of the permittivity reconstruction to a special geometrical representation of the objects. The Level set method was initially introduced for tracking the propagating boundaries. In this paper FEM has been used to solve the forward problem. In order to avoid so-called inverse crime, we used different mesh, triangular mesh for FEM model of the forward problem and a grid mesh for level set calculation. With an iterative method and using an update formula for level set function we try to fit the measured to the simulated data. We present a numerical implementation of 2D ECT reconstruction and the results of shape recovery are promising.

4 Methodology

ECT sensors measure the permittivity or dielectric constant of a sample. A typical ECT sensor comprises a circular array of 8 or 12 plate electrodes, mounted on the outside of a non-conducting pipe, surrounded by an electrical shield. The forward problem in ECT is the prediction of measured data for a given permittivity distribution and the electric potential on the electrodes. Pixel based image reconstruction in ECT is an inverse medium problem. There are several methods for image reconstruction in ECT. More commonly used image reconstruction for electrical imaging is based on the Born approximation. We start with an initial permittivity distribution. The forward problems are solved and the predicted capacitance compared with the calculated capacitance from the forward model. The permittivity is then updated using a regularized inverse of the Jacobian. The process is repeated until the predicted capacitance from the finite element method agrees with the measured capacitance. We use the linearized sensitivity of the capacitance measurement to a change in permittivity. This type of perturbation or Born approximation calculation will be familiar from the other linear inverse problems. Using pixel based reconstruction where we have two phase material is not an efficient method as it does not incorporate the known features of the image. Instead one can formulate the problem to find the interface between two materials. We have chosen to use the level set technique to describe the changing shapes, since this method is able to easily model topological changes of the boundaries. In the shape reconstruction approach, it is assumed that the background distribution and approximate values of the image parameter inside the inclusions are known, but that the number, topology and shapes of the obstacles are unknown and have to be recovered from the data. Compared to the more typical pixel-based reconstruction schemes, the shape reconstruction approach has the advantage that the prior information about the high contrast of the inclusions is incorporated explicitly in the modelling of the problem. Although in a pixel-based reconstruction scheme the approximate locations of the unknown obstacles are found already during the early iterations, it typically takes a large number of additional iterations in order to actually build up this high contrast to the background before getting more accurate information concerning the shapes of these objects. This can be done better with a level set approach. Here, the equation describing the moving fronts is

$$\Phi_t + F \nabla \phi = 0$$

(1)

where $F$ is the speed function and $\phi$ is the boundary in time $t$. In figure 1 we schematically show a moving boundary.
The describing level set function is a function form $R^2 \rightarrow R$ for this two-dimensional case, and its value is zero in the boundary, has a negative sign inside and a positive sign outside of the boundary. Figure 2 shows an important property of the level set function. When two parts of the shape merge or split, the level set function still can model the boundary accurately. So there is no need for specific adaptations of the model when the topology changes.

The inverse boundary value is to find a level set function (which in turn describes a conductivity or permittivity distribution) that minimizes the mismatch between the measured and simulated data. The algorithm is as follows:

1. Start with an initial guess for the shape of the inclusion (initial level set function), in our case a circle located in the centre.
2. Define the interface and narrow band; the narrow band is an area that includes pixels sharing points with the interface.
3. Solve the forward problem and calculating the Jacobian with respect to the boundary.
4. Update the level set function and calculating a new interface boundary and narrow band.
5. Checking the misfit in the data; if the error is small enough: stop.
6. If the misfit is not small go to step 2

For the forward problem we have implemented a FEM program. Figure 3 shows a typical ECT sensor, which includes the electrode array and the shielding.

The forward problem in ECT is an electrostatic problem where the conductivity is zero and the magnetic field is neglected. Assuming there are no internal charges, the mathematical model of the forward problem is given by

$$\nabla \cdot (\varepsilon \nabla \phi) = 0 \quad \text{in } V_d$$

$$\phi = \psi_k \quad \text{on } E_k$$

where $V_d$ is the region containing the field (possibly is an infinite region), $\varepsilon$ is dielectric permittivity $E_k$ is the $k$-th electrode, held at the potential $\psi_k$, usually attached on the surface of an insulator. The shields are set to zero volt. The capacitance between excitation and $k$-th sensing electrode is given by

$$C = \frac{Q}{V_0} = \frac{1}{V} \oint \varepsilon \nabla \phi \cdot nds$$

$$V_0$$ is the potential difference between the source and the detecting electrode; $Q$ is the total charge on the measuring electrode. In most practical ECT measurement the data are normalised as

$$\lambda = \frac{C_m - C_s}{C_s - C_i}$$

where $\lambda$ is the normalised capacitance between a pair of electrodes, $C_m$ is the measured capacitance,
$C_l$ and $C_h$ are the capacitances when the ECT sensor is full of the lower and higher permittivity materials used to calibrate the sensor respectively. We use finite element method (FEM) to solve the forward ECT problem. In first order triangular element the potential at each point inside of the element is

$$U = \sum_{i=1}^{3} U_i \alpha_i(x, y)$$  \hspace{1cm} (5)

Where $\alpha_i$ can be calculated based on coordinate data of the vertices of the triangle $((x_i, y_i, z_i), i=1,2,3))$, and $U$ is the electric potential at point $(x, y)$ and $U_i$ is the electric potential at node $i$ of the triangle.

$$\alpha_1 = \frac{1}{2d} ((x_2, y_2, -x_2, y_2) + (y_2, -y_2, x_2) + (z_2, -z_2, 0))$$  \hspace{1cm} (6)

$$\alpha_2 = \frac{1}{2d} ((x_3, y_3, -x_3, y_3) + (y_3, -y_3, x_3) + (z_3, -z_3, 0))$$  \hspace{1cm} (6)

$$\alpha_3 = \frac{1}{2d} ((x_1, y_1, -x_1, y_1) + (y_1, -y_1, x_1) + (z_1, -z_1, 0))$$  \hspace{1cm} (6)

And each element matrix (given a piecewise constant value of the permittivity $\varepsilon_i$ in each finite element) is an integral over the area of the element

$$K_{ij}^{e} = \int \varepsilon_i \nabla \alpha_i \cdot \nabla \alpha_j dS$$  \hspace{1cm} (7)

The linear system of equation $K\nu=b$ can be solved using a preconditioned conjugate gradient (PCG) method. Here $b$ is the term related to the boundary condition. A typical ECT mesh can be seen in Figure 4.a.

Figure 4.b shows the electric potential distribution when the electrode 1 was used for the excitation for empty tank.

The sensitivity of the change in measured charges on sensing electrodes to the change of the permittivity of a region in imaging area calculated by:

$$\frac{dQ_{ij}}{d\varepsilon} = \int_\Omega \delta\varepsilon E_i \cdot E j dS$$  \hspace{1cm} (8)

Where $E_i, E_j$ are the electric field when the excitation electrodes are electrodes $i,j$ that can be calculated directly from the results of the forward solver. $\Omega$ is the perturbation region. Figure 4.c shows the sensitivity plot for two opposite electrodes.

A Regularised Gauss-Newton method is a suitable image reconstruction scheme for non-linear ill-posed ECT problem. The image reconstruction is to find a permittivity that minimises

$$\left\|C_m - F(\varepsilon)\right\|^2 + G(\varepsilon)$$  \hspace{1cm} (9)

where $C_m$ includes the measured capacitances and $F(\varepsilon)$ is the simulated capacitances for the permittivity map $\varepsilon$ from the forward problem, and $G(\varepsilon)$ is the regularisation term.
For a shape-based method, using level set formulation the boundaries are moving in order to reduce the misfit errors. Figure 5 shows a perturbation of level set function in each iteration of shape identification problem.

5 Experimental results

In figure 6, the reconstruction of a ring shape object with relative permittivity 1.8 in a background with relative permittivity 1 is shown with traditional image based as well as level set shape. The experimental data was measured using an ECT system.

![Image](a)

![Image](b)

![Image](c)

Figure 6: True object (a), reconstructed image (b) and reconstructed shape (c)

The level set function could find the position and a relatively accurate shape of the inclusion; however more work needed to be done in regularization of this method in order to improve the shape recovery. The result of level set is comparable with that of the image based and the computation time for the shape method is less.

6 Conclusion

Shape identification is an inverse boundary value problem; therefore it is not efficient to use the common image reconstruction methods. Shape reconstruction with real experimental ECT data presented in this paper. The main advantage of the level set formulation is that in each iteration the inverse problem needs to be solved in the interface between two materials rather than all region of interest. In terms of including all prior information, level set method incorporate and important regularization, namely two-phase material.

References


